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Ultrasound Computer Tomography in Diffraction Mode

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Abstract

Ultrasound computer tomography aims at safe and fast high resolution imaging. One can imagine Ultrasound computer tomography (USCT) as an imaging procedure where X-rays in a CT scanner are replaced by Ultrasound waves, but unlike imagination the X-ray imaging principle cannot be directly applied because ultrasound does not travel in a simple straight line alone. It undergoes diffraction due to relatively large wavelengths associated with typical ultrasound sources. USCT in diffraction mode tomography uses an alternate approach known as inverse scattering problem for reconstructing the parameters of interest. In this paper, the wave equation is theoretically and numerically solved considering wave as a function of compressibility and velocity. The received field found by solving wave equation was simulated. These experimental results indicate that derived method can yield images with higher image resolution in a strong scattering field.

Keywords: *Ultrasound Computer Tomography, Diffraction Mode*

1. Introduction

Ultrasound computer tomography (USCT) as the name implies uses ultrasound for sliced images and then reconstructs back into detailed image for better diagnosis. In general USCT components include ultrasound transducers (Single transducer or array of transducer) that can be arranged in ring shape or linear fashion or free handed too. These transducers transmit sound waves and also receive the sound waves back in form of echoes. Hence transducer can be used as either transmitter or receiver. The other general components include a pulse generator, amplifier, digital oscilloscope and computer. The architecture of USCT varies according to the transducer arrangements. There are several ways of arranging the transducers based on one's requirements. As shown in the figure, transducers in the USCT systems can be possibly arranged opposite to each other or in a linear fashion or in ring shaped manner [1]. Fig 1.a. shows a simplified model of ultrasound tomography which has two transducers that can be rotated readily around the object. Fig.1.b. Here transducers are arranged in ring shaped array enclosing the object. All the transducers are marked as T/R, which implies these can be used as transmitter or receiver. Other form of arrangement is represented in Fig.1.c. in which the transducers are aligned as two linear opposite array rotating mechanically around the object

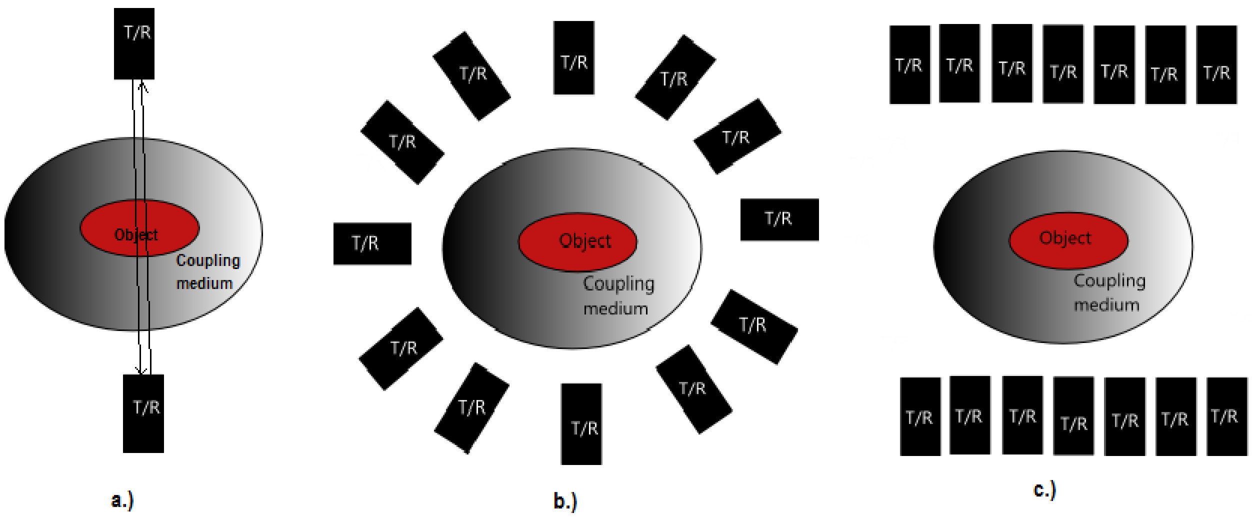

Fig.1. Arrangement of transducers in USCT

Based on the propagation of ultrasound in the object of interest it can be classified into three modes, transmission mode, reflection mode and diffraction mode. When Size of Objects is much smaller than the wavelength of ultrasound, ray statistics cannot be applied. Hence in case of ultrasound study, diffraction theory is necessary. USCT working in diffraction mode can be explained using a model in Fig.2. In this, let us assume the system works on single transducer and the object of interest are placed at a distance from the transducer. The transducer acts as both transmitter and receiver. When the transducer is excited by voltage, the object is insonified by ultrasound transducer forming a wide cone shaped beam. In the cone-beam case, circular-arc wave fronts are produced, while, in the plane-wave case, the wave fronts consist of parallel straight lines.

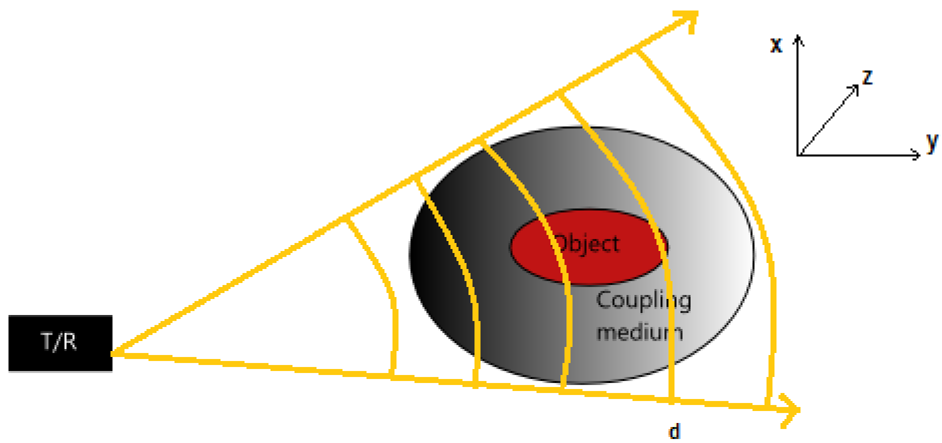

Fig.2. USCT in diffraction mode

Diffraction tomography is similar to reflection tomography where instead of summing all the received signals, each one separately recorded and reconstructed taking the Fourier transform of each received signal with respect to time. Much recent advancement has brought a new dimension for diffraction tomography on a strong scattering field and produced higher image quality under different boundaries [5] [6].

Iwata and Nagata [2] were first to bring in the idea of ultrasonic tomography in diffraction mode where they calculated refractive index distribution of object of interest using the Born and Rytov's approximation. Later, Mueller et al. and Stenger et al. investigated on this method through various reconstruction algorithms [3] [4].

When Ultrasound penetrates through an inhomogeneous medium, it undergoes diffraction and creating a scattered field in the output. The characteristic of the scattered field reveals the tissue property. This scattered field forms the base of the ultrasound diffraction tomography. The essential components of ultrasound diffraction tomography are, firstly, a mathematical model describing the tissue-sound interaction and parameters of interest. Secondly,a reconstruction procedure to deduce the parameter of interest from the measured solutions.

2. Wave equation derivation

The mathematical model in Ultrasound diffraction tomography is derived by solving wave equation. Here we use an alternate approach known as inverse scattering problem for reconstructing the parameters of interest. The direct scattering theory and inverse scattering theory fall under scattering problem theory. The direct scattering theory is to determine the relation between input and output waves based on the known details about the scattering target. The inverse scattering theory is to determine properties of the target based on the computed input-output pairs. Tomographic reconstruction in diffraction mode uses inverse scattering approach also known as forward scattering problem. Commonly scattering problem is solved with Lippmann-Schwinger or Integral equation using Green's function.

Consider an object with density ρ_o is insonified by ultrasound wave in the Cartesian coordinates, and then the reconstruction is mapped by solving the forward scattering problem with the linear approximation. Let us assume, the spatial gradient of sound pressure wave in Cartesian coordinates represented as $\nabla p(r,t)$ in the point r in the space, then Homogeneous acoustic wave equation is represented as;

$$\nabla^2 p(r,t) - \frac{1}{C2}\frac{\partial 2\, p(r,t)}{\partial t2} = 0 \qquad (eq1)$$

C is the speed of sound in the medium. Applying Fourier transform to the above equation, we get Helmholtz equation that can be written as;

$$\nabla^2 p(r,w)+k^2 p(r,w)=0 \qquad (eq2)$$

where *k* is the acoustic wave number such that $k=w/c= 2\pi/\lambda$. But practically a human body is an inhomogeneous medium. As discussed earlier when ultrasound penetrates through human body it scatters. Let the scattering term be *f(r,t)* due to the scattering object at point *r'* at time *t'*. Therefore, the wave equation can be rewritten as,

$$\nabla^2 p(r, t) - \frac{1}{C2}\frac{\partial 2\, p(r,t)}{\partial t2} = -f(r',t') \qquad (eq3)$$

We assume the scattering is due to change in density and compressibility of the medium. Wave equation for inhomogeneous medium with scattering field in terms of density and compressibility was derived as [Morse and Ingard, 1968],

$$\nabla^2 p(r, t) - \frac{1}{C2}\frac{\partial 2\, p(r,t)}{\partial t2} = - \frac{1}{C2}\gamma(r)\frac{\partial 2\, p(r,t)}{\partial t2} + \nabla(\Delta\rho).\nabla P \qquad (eq4)$$

$\gamma(r)$ is the change in compressibility and $\Delta\rho$ change in density. Using first-order Born approximation, the scattered signal is calculated using Green's function. Employing Green's function *g(t)* for unbounded space which is,

$$g(r, t \mid r', t') = \frac{1}{4\pi|r-r'|}\,\delta\left(t-t'-\frac{|r-r'|}{c}\right) \qquad (eq5)$$

The pressure field in the point r' in the space is calculated as,

$$p(r, t) = \int_V \int_t f(r', t') * g(r, t \mid r', t') \qquad (eq6)$$

From *(eq4)* and *eq(6)* the measured scattered pressure field p_s (r, t) can be calculated as

$$p_s(r, t) = \int_V \int_t - \frac{1}{C2}\gamma(r)\frac{\partial 2\, p(r,t)}{\partial t2} + \nabla(\Delta\rho).\nabla P * g(r, t \mid r', t')\, dv\, dt$$

For a computer simulation the scattered pressure field $p_s(r)$ in the region interest is digitized into NxN square pixels when applied by an incident pressure field $p_i(r)$ is given as,

$$p_s(r)=p(r)- p_i(r) \qquad (eq7)$$

The incident field is calculated as;

$$p(r,t) = \rho_o \frac{\partial v(t)}{\partial t} * h(r,t) \qquad (eq8)$$

where *v(t)* is the velocity waveform and *h(r,t)* is the spatial impulse response of the system. Hence the received signal is the scattered pressure field integrated over the line of integral (transducer surface) convoluted with the electromechanical impulse response. Now we know using Born's approximation, the scattered field is solved. Then solution would be,

$$p_s(r) = \iint o(r)\, p_i(r)\, g(r,t|r',t')\, dv\, dt \qquad (eq9)$$

where $o(r)$ represents the scattering term. Considering for a delta function as the test function, the measured field at any point in the medium is,

$$p_s(r) = \sum_j o(r)\, p_i(r) \int g(r,t|r',t')\ \delta(r-r')dr'dt \quad j=1,2,3\ldots N$$

So, $$p_s(r) = \sum_j o(r)\, p_i(r) \int g(r,t|r',t')\ dr' \qquad (eq10)$$

Using Hankel function, the 2D green's function can be solved. According to Hankel function properties, we know that for scattering point source *r'* of radius *a* and distance (*R*) between *r* and *r';*

$$\int g(r,t|r',t')\ dr' = \begin{cases} \frac{j}{2}\ [\pi ka H^2(ka-2j)] & for\ i=j \\ \frac{j\pi ka}{2}\ J(ka)H^2(kR) & for\ i \neq j \end{cases} \qquad (eq11)$$

K is the wave number, J is the first kind of bessed function and *H* is the second class of first order Bessel function. On applying (eq11) on (eq10), the receiving scattering field due to scattering point source r' can be transformed into,

$$p_s(r) = \sum_j o(r)\, p_i(r)\, d_j \qquad \text{where } d_j = \frac{j\pi ka}{2}\ J(ka)H^2(kR) \qquad (eq12)$$

The total pressure field can be calculated as,

$$p(r) = p_i(r) + \sum_j o(r)\, p_i(r)\, d_j \qquad (eq13)$$

The inverse scattering problem which is established based on wave theory can be ill posed equation. The method to solve these ill-posed problems is use of regularization. The scattering term can be estimated using Tikhonov regularization as where γ is the regularization parameter.

$$\overline{\boldsymbol{O}} = arg\ \min_{O} \|\overline{\boldsymbol{p}} - \overline{\boldsymbol{M}}\overline{\boldsymbol{O}}\|_2^2 + \gamma\|\overline{\boldsymbol{O}}\|_2^2 \qquad (eq14)$$

O(r) was updated with each iteration by Tikhonov regularization.

3. Simulation output:

Simulated data were created for 3D muscle model with 256x256x256 voxels and the medium was set using FOCUS software. The scattering field is affected by 5% Gaussian noise. A number of transducer ultrasound transmitters are positioned around the object in the ring configuration as in fig1.b. We simulated the number of transducer surrounding the object to be 76. The computation was performed on a basic system equipped by Intel Core i3 (2.1GHZ, 3Mb l3 Cache) and 750 GB HDD. It produced immediate results in 7.85 seconds which is a faster process time for simulation. Figs. 3 and 4 is the simulated pressure field and expected pressure field. The simulated data is a true match with expected data.

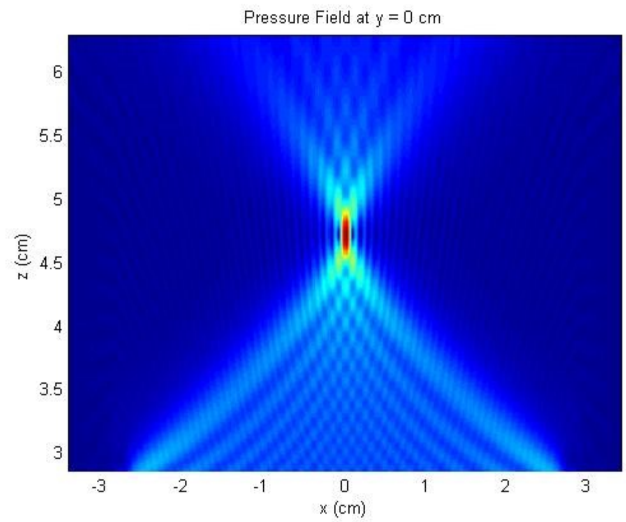

Fig.3. Simulated scattered pressure field plotted for muscle

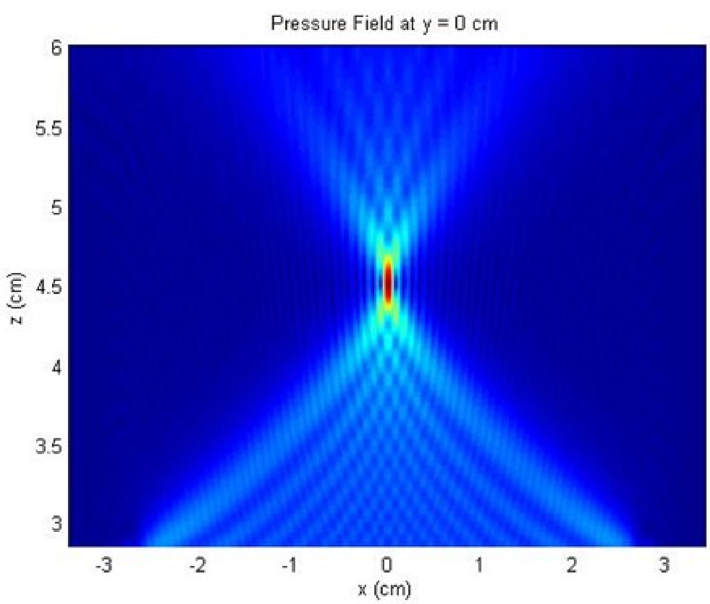

Fig.4. Expected measured scattered pressure field.

4. Conclusion and future work:

Simulation results indicate that derived method can yield images with higher image resolution in a strong scattering field. Using Tikhonov regularization, the computation iteration was decreased and can greatly enhance the quality of reconstruction. As the computation complexities ease decreased with computing scattering term using Tikhonov regularization, we have a scope for new and easy ways of implementation. Our future work includes testing the accuracy and performance with other methodologies. Hence, there is a huge scope for future advancements in 3D ultrasound diffraction computer tomography. The 3D-USCT has the potential to extend its applications in areas like diagnosis of testicular cancer, diagnosis of hip dysplasia in newborns and search for inclusion and defects in bulk materials. As the sound needs a medium to travel, this ultrasound needs a coupling medium such as water or gel to transmit the ultrasound into the human body. So this coupling medium restricts the use of ultrasound to a smaller region because it isn't practical to couple whole human body with any coupling medium. Henceforth Ultrasound computer tomography is more devoted to the diagnosis of benign and malignant tumors in soft and small biological tissues. The future work also encourages applying Ultrasound computer tomography for larger areas with the noticeable advancements in terms of software and hardware.

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