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Frequency analysis of anomalous negative price fluctuations in stock market indices as a crisis forecasting tool

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Abstract

Stock and bond markets are inherently forward-looking since they anticipate the future earnings of a company and thus the future state of an economy. Hence, they are the go-to recession forecasting indicators for retail and professional investors. Composite indexes reflect the condition of the general market or its sector. The efficient market hypothesis, proposed by Fama in 1965, is the central argument in describing the behavior of composite stock market indices and their data generating process. Applying non-parametric statistical analysis to a composite stock indexes' log-return distribution function one could derive not only the market sentiment but also measure the level of volatility and distress in the economy. When the price of a stock market index moves below 2.33 deviations from the mean, it is considered an anomaly, meaning its occurrence is extremely unlikely. Yet many investors tend to underestimate the likelihood of such an occurrence and overlook its meaning. In this research, we aim to introduce new methods of determining when an economy is in a recession through anomalous frequency analysis of a countries' domestic composite index. We also aim to present the algorithm and provide empirical evidence of the theory on the Hungarian and Polish markets. Capital markets provide real-time insight into the economy of a nation. While some leading economic indicators, such as retail activity, the housing market, manufacturing activity, and changes in GDP, are valuable in detecting imbalances in an economy, perhaps no other indicator is as important or as popular as the indicators based on the stock market.

Keywords: crises, indicators, stock market, algorithm

1. Introduction

The Efficient Market Hypothesis (EMH) predicts that it is impossible to consistently pick stocks that outperform the market. Therefore, beating the market is rather a matter of chance than actual skills. Timmerman and Granger noted that asset price returns are unknowable and unforecastable. They base it on the logical conclusion that if returns were, to some degree, be forecastable within a confidence interval, investors would use these forecasts to generate unlimited profits (Timmerman and Granger, 2004, p. 15). Nevertheless, many studies point out that there exists a correlation between economic activity and stock market returns. The earliest of such observations conducted by Fama (1981) involved the negative correlation of stock returns and inflation. Fama found that the industrial production variable holds the strongest relationship with stock returns. And while both Fischer (1984) and Moore (1983) studied the effects of policy and exogenous variables on stock returns, their data, reasoning, and calculations should be subject to review. This is because the nature of the stock market changed over the decades. Since, there is a relationship between current market valuations and expected future returns, timing the market should work as a consistent profit-generating strategy. When stocks are expensive relative to past valuations, future returns are expected to be lower. There is unquestionable evidence that in the past fifty years stocks have been getting more and more expensive for longer periods, therefore rendering the market timing strategy useless (Market timing…, 2017). Market deregulation, the emergence of complex speculative derivatives, and products not requiring the possession of the underlying asset have altered the price-action of securities and made markets unstable.

There have been many policy measures enacted by governments to prevent financial crises from happening and to avert the growth of corporations to the extent of becoming "too big to fail". The Basel III accord raised banks' minimum capital requirement and risk exposure, set a backstop leverage ratio, and introduced liquidity requirements – thus tightening the regulation of the financial sector on an international level. On national levels, the US has introduced the Dodd-Frank and consumer protection act, while European monetary institutions introduced alert mechanism reports and early warning systems (EWS), which give an early insight into emerging macroeconomic imbalances. Due to a high degree of decentralization, another "Lehman Brothers" scenario in the current economy is highly unlikely. Notwithstanding the successful implementation of policies around the globe, macroeconomic shocks as a result of unexpected occurrences referred to as "black swan" events are inevitable and practically impossible to foresee. Neither pandemics nor liquidity crises or rapid currency devaluations are predictable, much less their effect on the economy. Therefore, EWSs serves as a critical framework for policy action and the timely detection of macroeconomic imbalances. The monitoring of individual economic indicators for intersecting crisis-signaling threshold values determined through the signal method (Kaminsky, Lizondo, and Reinhart, 1996), has been the subject of previous research on the topic of EWSs, including for the Hungarian market (Csortos, Szalay, 2013).

The stock market has often been viewed as a predictor of a country's economy. If stock prices show anomalous negative returns, it is a signal that economic growth stopped, and a recession is on the way. Specific indicators, like the spread between treasury bonds or stock market valuation to GDP ratios, have a good history of correctly predicting recessions (Williams, 2021). In this paper, we develop a similar indicator that uses the frequency of quarterly negative composite stock index price fluctuations to predict a recession. The relationship between stock returns and economic growth was first researched by ( ). The widely debated question of stock markets' capability of predicting recessions is viewed from the scope of Pearce's (1983) paper on stock prices and the economy. Based on Comincioli (1995), it has been determined that composite stock market indexes Granger cause changes in the economy. For simplicity's sake, this research omits the criticism surrounding Comincioli's method and findings. The question the paper addresses is whether stock market fluctuations could be used as an efficient economic crisis forecasting tool? Can market anomalies be interpreted outside of a statistical framework? The paper aims to outline a crisis prediction model by determining the algorithm for frequency analysis of anomalous negative price fluctuations in composite stock market indexes. The paper specifically targets emerging markets as a means for conducting the empirical analysis, as we assume the indicator works better for emerging markets rather than developed ones.

The rest of the paper is structured the following way: first, we discuss the theory of EMH, its criticism, and its role in our model. In the material and method section, we present the algorithm and the calculations for determining the number of anomalous price fluctuations of a composite stock market index. The empirical results section applies the algorithm to the Hungarian, Ukrainian, Russian and British composite stock market indexes to explain its efficiency in emerging markets.

1.1.The efficient market hypothesis and its criticism

No model is a perfect reflection of reality. At the core of any market is the logic of asset pricing – how do market participants decide how much to pay for a security. As formulated in (Sharpe, 1977), the market value of an asset is the present value of its expected future cash flows. Regardless of the rationality of the investors' asset evaluation process, there exists a theoretical framework for an "ideal" market. In his groundbreaking work, through a review of the theoretical and empirical literature, Eugene Fama developed the theory of an efficient market, which is a model where market prices "fully reflect" all available information. To quantify the model, Fama uses the fair game, sub martingale, and random walk methods to express the expected return of an asset as a function of risk. This puts the theory to the test within a statistical framework. In addition to the empirical work, Fama introduces three conditions for which markets can be deemed informationally efficient: (i) information is freely available to market participants; (ii) there are no transaction costs in securities trading; (iii) there is a competition for profits. (Fama, 1970, pp: 385-387) So far, practice shows that at least two of the aforementioned conditions aren't met by real-life markets. This is supported by incidents of insider trading, existing brokerage transaction fees, and the presence of market bubbles. Further interpreting the theoretical framework of the EMH, it is evident that Fama treats the flow of information in markets as a random stochastic event, and therefore deems the stock market to be inherently unpredictable. It can be reasoned that, since a significant part of the price action of a security is attributed to the flow of information, which in itself is unpredictable, therefore by trying to predict the price action as an active manager, will not improve the outcome of an investment. Samuelson agreed that should investors know the exact future price-action of a security, then its further behavior would be altered by this information, which has been instantly incorporated into the price – supporting the claim that properly anticipated prices fluctuate randomly (Samuelson, 1965, p 13) Unlike Fama, Samuelson uses a deductive approach and dismisses the hypothesis that a price of an asset follows a pure Brownian motion or a random walk. However, what neither of the Nobel Prize-winning economists considered is whether the random walks of stock indexes could be attributed to larger macroeconomic trends? The EMH states that the nature of the price-action of a stock is random and therefore unpredictable. This allows for a statistical framework to be built around it. In particular, since the log-returns of a stock can be graphed in a normal distribution, we can observe the rightmost and its leftmost values. And by declaring a threshold level, determine those values, which are anomalous. A stock return, which is considered anomalously negative, can therefore be traced back to a date at which some correlation could be found with economic output at that particular period.

2. Material and method

The efficient market hypothesis (EMH) proposed by (Fama, 1965) is the central argument in explaining the behavior of composite stock market indices and their data generating process

(DGP). The processes of dissemination and interpretation of these signals by market participants may be distorted by market anomalies, questioning the postulates of the EMH. Among the key anomalies, (Plastun, Makarenko, 2018) draw our attention to spikes in volatility, seasonal anomalies and an unpredictable change in profitability under the influence of a range of market variables. Anomalous price fluctuations of indices on the stock market can be used as a separate block of leading indicators for forecasting economic crises on the line with traditional macroeconomic indicator. Notwithstanding the significant diversity of warning signals on the stock market, the search for new methods continues to this day. The role of traditional macroeconomic indicators such as GDP growth, industrial production indices, inflation, unemployment rate in forecasting economic crises is often neglected, due to their general low predictive capabilities, and a significant lag in their publication. However, there is an array of economic indicators, devoid of these shortcomings, the access to which is available in real time for all users. Among these indicators, the most prominent are composite stock market indices containing a weighted basket of stocks, others are variables that characterize the condition of the stock market, for instance the RTVX- which in itself is a volatility index. In this context, forecasting of crisis phenomena in the economy can be done using various parameters and indicators of the stock market: volatility, correlation of stock market assets, its long-term memory (persistence), frequencies of abnormal price fluctuations (overreactions and underreactions). Empirical studies on stock market price fluctuations inter alia (Sandoval, Franca, 2012; Stefanescu, Costel, 2012) outline that a sudden increase of the number of anomalous negative fluctuations in the stock market could indicate that the economy of the country is about to enter a recession. The hypothesis put forward by (Plastun, Makarenko, 2018, p. 74) states that the increase in the frequency of anomalous negative fluctuations of the stock index indicates the intensification of crisis phenomena. Accordingly, the phases of crisis development correlate with the dynamics of changes in the frequency of anomalous negative fluctuations. Adopting the notation of (Sandoval and Franca, 2012), it becomes evident that the goal of the method is the creation of a visual graph that displays the frequency of anomalous variable fluctuations on a monthly, quarterly or yearly interval. The variables of the stock indicator are normalized, so they are between -1 and 1 – this makes it easier to compare variations of the time indices. The expression for the log-returns is given by the following expression:

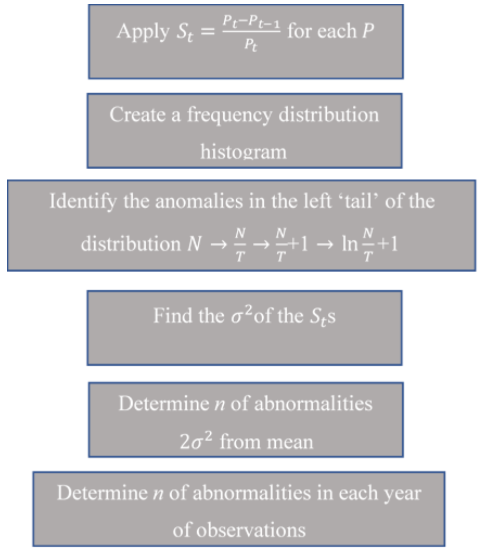
$$S_t = \frac{P_t - P_{t-1}}{P_t},$$

also expressed as

$$S_t = \ln(P_t) - \ln(P_{t-1})$$

The log-density distribution function is graphed, and a certain number of the leftmost outliers are counted for each given time period. To simplify the above described algorithm, Figure 1 illustrates the procedure on a step-by-step basis. The explanations of the expression above are detailed in the empirical results.

Figure 1.: The algorithm for determining anomalous price fluctuations.

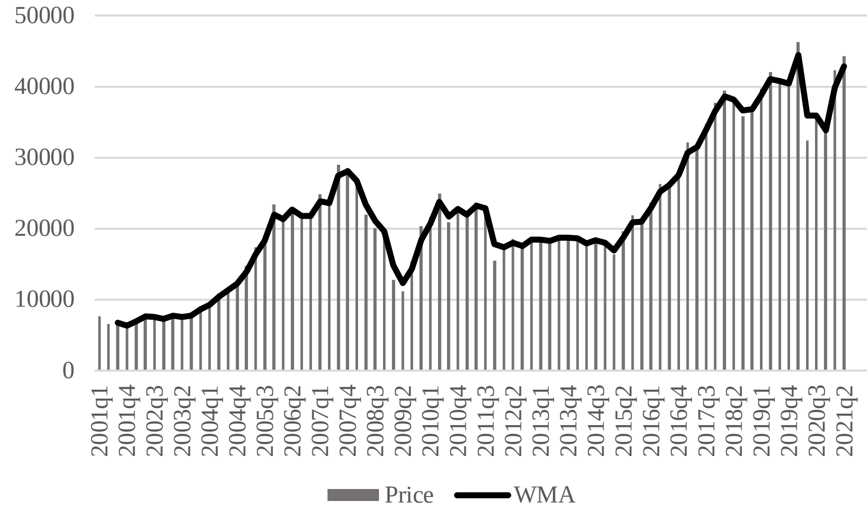


Source: own authoring

2.1.Empirical results

As to demonstrate the effectiveness of the method on Hungarian stock markets, the BUX stock market index was analyzed within the anomalous fluctuation frequency analysis framework. In this research we looked at the daily closing price of the index from 2001 until 2021 as seen in Figure 2. From a visual inspection of Figure 3, it can be seen that there exists some sort of correlation between the log returns of BUX and the quarterly percentage change of the Hungarian GDP. The data was gathered from the MNB (Hungarian National Bank). Altogether, about 5056 values were obtained. For the sake of simplicity of representation, the data on Figure 2 is quarterly. The expression: $S_t = \frac{P_t - P_{t-1}}{P_t}$ states that for each subsequent days' closing price, the value of the previous days' closing price is subtracted and divided by the subsequent days' closing price, where P_t is the closing price value of the index. t – denotes the moment when the value of closing price of the index was recorded and P_{t-1} is the previous days' closing price.

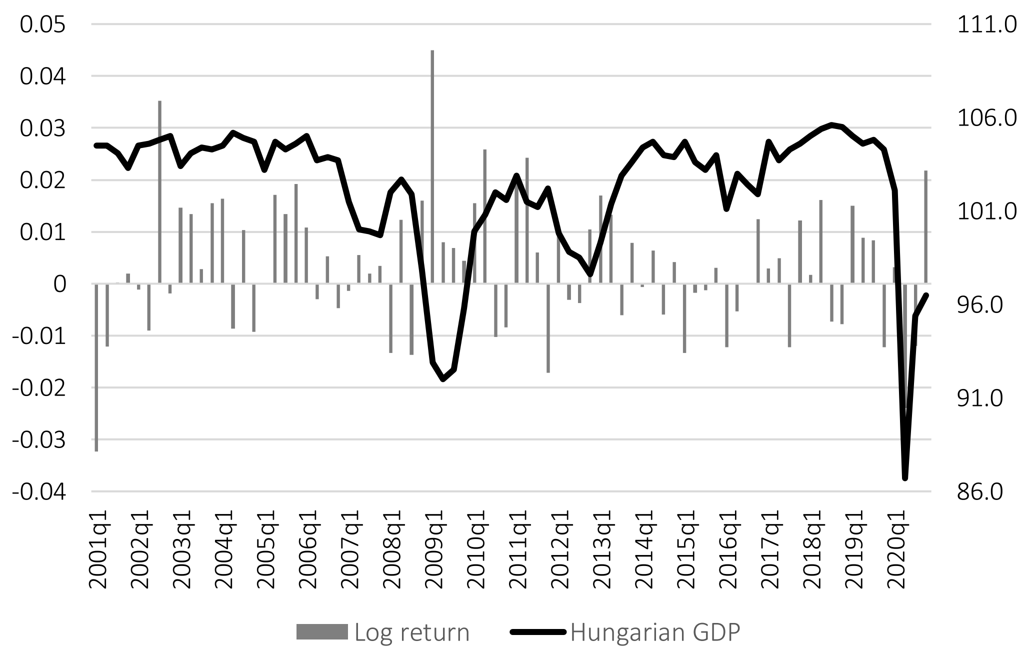
Figure 2.: BUX time series with Weighted moving average.



Source: MNB, Compiled by the author

What can be concluded from Figure 3, is that periods of rapid GDP decline are followed by large jumps in BUX log returns. Evidently, this could be attributed to specific market behaviour that occurs when the discount to the fair value of a stock seems attractive enough for investors.

Figure 3.: Hungarian GDP vs BUX log return.



Source: own compilation based on MNB

This naturally drove us to conduct further analysis on the time series. We checked whether the BUX log returns Granger causes the GDP percentage change. To test this, we first had to make sure that the two time series are stationary.

Table 1.: Augmented Dickey-Fuller tests for BUX log returns and GDP percentage change

ADF Statistics: -11.556436 p-value: 0.000000 Critical values: 1%: -3.496 5%: -2.890 10%: -2.582	ADF Statistics: -9.379186 p-value: 0.000000 Critical values: 1%: -3.495 5%: -2.890 10%: -2.582
Augmented Dickey-Fuller test for GDP percentage change	Augmented Dickey-Fuller test for BUX log returns
Results of KPSS Test: Test Statistic 0.110045 p-value 0.100000 #Lags Used 4.000000 Critical Value (10%) 0.347000 Critical Value (5%) 0.463000 Critical Value (2.5%) 0.574000 Critical Value (1%) 0.739000 dtype: float64 Result: The series is stationary	Results of KPSS Test: Test Statistic 0.309418 p-value 0.100000 #Lags Used 2.000000 Critical Value (10%) 0.347000 Critical Value (5%) 0.463000 Critical Value (2.5%) 0.574000 Critical Value (1%) 0.739000 dtype: float64 Result: The series is stationary
KPSS test for GDP percentage change	KPSS test for BUX log returns

Source: own compilation

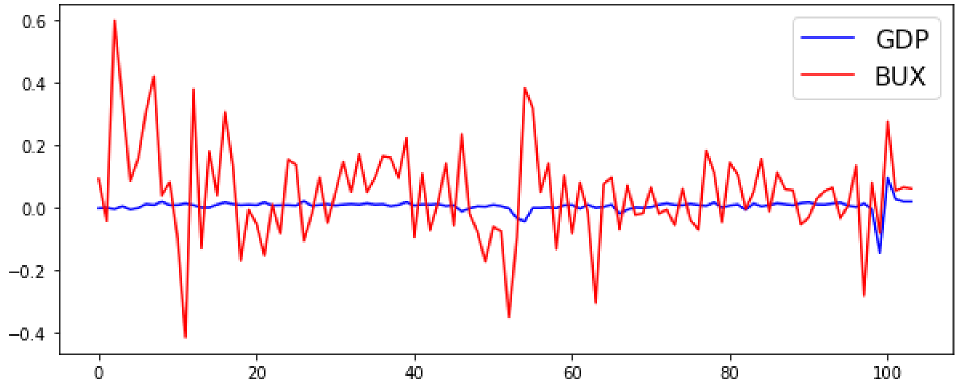
To test the stationarity of the time series in Python we imported the augmented Dickey-Fuller test from the ‘statsmodels’ library. The listing of the program is provided in the Appendix.

Since the P-values in both cases are smaller than the 0.05 significance level, it can be concluded that both of the time-series are stationary, which means their means are constant over time. A KPSS has also been applied and the output of the program was that both time series are stationary. This means that we are free to proceed with the rest of the Granger-Causality analysis. The next step is finding out whether the GDP time series is actually Granger caused by the BUX time series. For this, we import the ‘grangercausalitytests’ to Python together with the ‘pandas’ extension. We also have to adjust both time series in a manner, so that one of them is lagged. We achieve this in Python with the following code:

```
BUX = BUX[1:]
GDP = GDP[:-1]
```

Since the data are in quarters, the code above shifts the BUX time series ahead of the GDP by 1 quarter also displayed on Figure 4.

Figure 4: Shifted time-series.



Source: own compilation

We utilize the 'pandas' extension to arrange the shifted data in a table, so that the Granger causality test can be performed.

```
ts_df = pd.DataFrame(columns=['BUX', 'GDP'], data=zip(BUX,GDP))
gc_res = grangercausalitytests(ts_df, 3)
```

We obtain the final results in the Table 2 below.

Table 2.: BUX and GDP Granger causality test at 3 lags.

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=0.4029  , p=0.5271  , df_denom=100, df_num=1
ssr based chi2 test:   chi2=0.4150  , p=0.5195  , df=1
likelihood ratio test: chi2=0.4141  , p=0.5199  , df=1
parameter F test:         F=0.4029  , p=0.5271  , df_denom=100, df_num=1
```

```
Granger Causality
number of lags (no zero) 2
ssr based F test:         F=2.6985  , p=0.0724  , df_denom=97, df_num=2
ssr based chi2 test:   chi2=5.6753  , p=0.0586  , df=2
likelihood ratio test: chi2=5.5230  , p=0.0632  , df=2
parameter F test:         F=2.6985  , p=0.0724  , df_denom=97, df_num=2
```

```
Granger Causality
number of lags (no zero) 3
ssr based F test:         F=2.0651  , p=0.1101  , df_denom=94, df_num=3
ssr based chi2 test:   chi2=6.6567  , p=0.0837  , df=3
likelihood ratio test: chi2=6.4466  , p=0.0918  , df=3
parameter F test:         F=2.0651  , p=0.1101  , df_denom=94, df_num=3
```

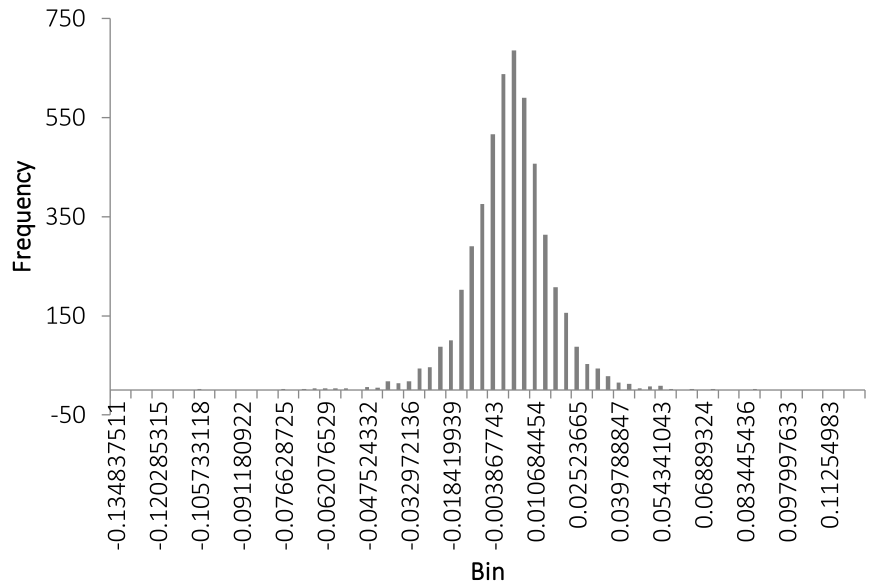
Source: own compilation

Even at three lags, the p values barely cross the 0.1 significance level, which is not considered to be enough to reject the null hypothesis that the BUX values do not predict GDP percentage change. Still, taking into consideration the achieved results, we can still proceed with the anomalous price fluctuation frequency analysis.

When we compute S_t for every P, we use a frequency histogram to visualize the optimal binning of the intervals and determine the outliers in the leftmost tail of the distribution. In order to smoothen the curve, a logarithmic transformation can be applied given by $\ln(\frac{N}{T}+1)$, where N is the frequency of the observed values in a bin, and T is the total number of observations, which in our case is 5056. This step can be omitted, as it doesn't add nor subtract any valuable information from the process. The subsequent steps of defining the left tail threshold can be executed using the histogram. In Figure 12 the frequency histogram for the

BUX index is graphed. Figure 13 features the log-density distribution function. Empirical studies point out that the log-density function t is used to visualize the most extreme points better. Although in case of our research, it smoothed them out. The purpose of the visualization of the histogram or the log-density function is to visually determine the position of the threshold relative to the mean. The equations given in (Sandoval and Franca, 2012) and (Plastun, Makarenko, 2018) define the threshold level as "the 10 most negative variations", which means that the leftmost 10 bars are used for further measurements. No statistical, or logical background as to the choice of the threshold have been offered by either of the authors – this is a major pitfall in the methodologies. In this research paper, instead of giving a constant for determination of the threshold, we instead use the threshold value of $2\sigma^2$ from the mean. The logical reasoning behind this decision is that only 2.5% of the observed data will be featured in the final frequency graph.

Figure 5: Frequency distribution histogram of the S_t of the BUX index.



Source: own compilation

To calculate the standard deviation, we used an Excel function the result of which was 0.014. In the final frequency graph, we therefore included all of the outliers below -0.028 and determined how many of these anomalies occurred in a given year of observations Table 15. In Figure 14 the results were graphed.

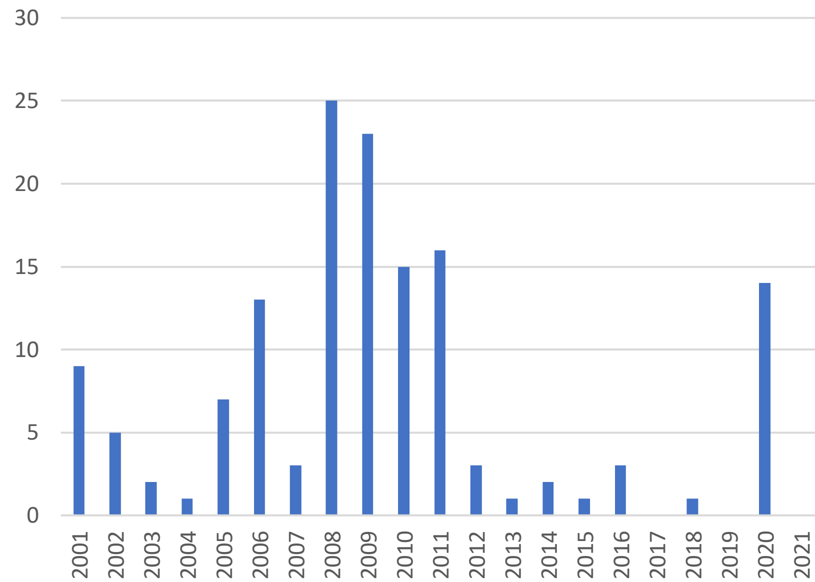
Table 3.: Frequency of anomalous BUX price fluctuations yearly.

2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
9	5	2	1	7	13	3	25	23	15	16	3	1	2	1	3	0	1	0	14	0

Source: own compilation

At this point a frequency histogram is useful to graphically represent the table above. Following the algorithm defined in Figure 1, upon close inspection of the histograms, we need to conduct a qualitative assessment of the peaks and recessionary periods in the economy. In Hungary, for instance such periods are associated with the following years: 2006, 2008, 2013 and 2020. Visualized in Figure 6, we can see peaks in the years listed before. The peaks are interpreted as the numbers of anomalous price fluctuations in a given year. In 2007, for instance, we can observe that out of the 253 trading days in 35 the stock index returns were so negative that their likelihood of appearance under normal market conditions is so rare that it is practically statistically negligible. The threshold level of the distribution, of course, plays a major role in determining the number of anomalous occurrences.

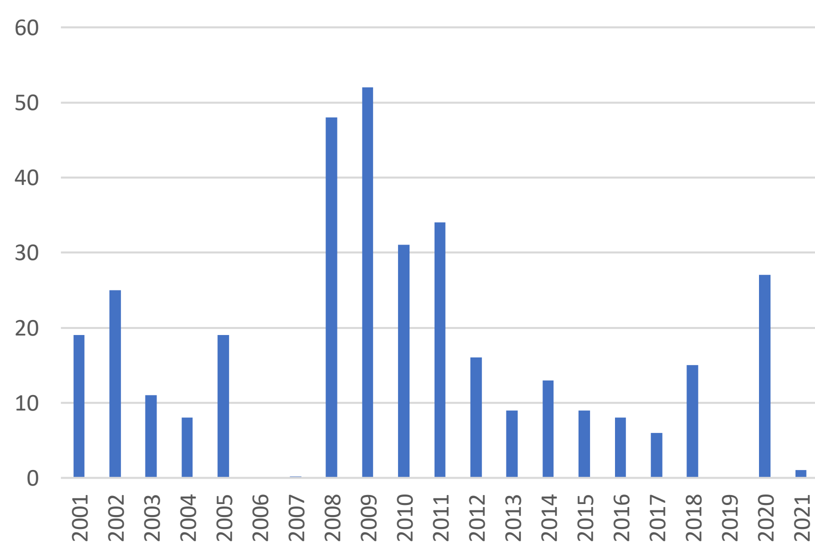
Figure 6.: BUX anomaly frequency chart at -0.028 threshold.



Source: own compilation based on MNB

We also tested the claim by (Plastun and Makarenko, 2018 p 74) that the threshold point must be in the lower 2-3% of the observed data – the threshold in Figure 7 was -0.018. If we turn to the histogram in Figure 7, we can observe that the binning value of -0.018 also corresponds to the 10th bar of most negative variations, following (Sandoval and Franca, 2012).

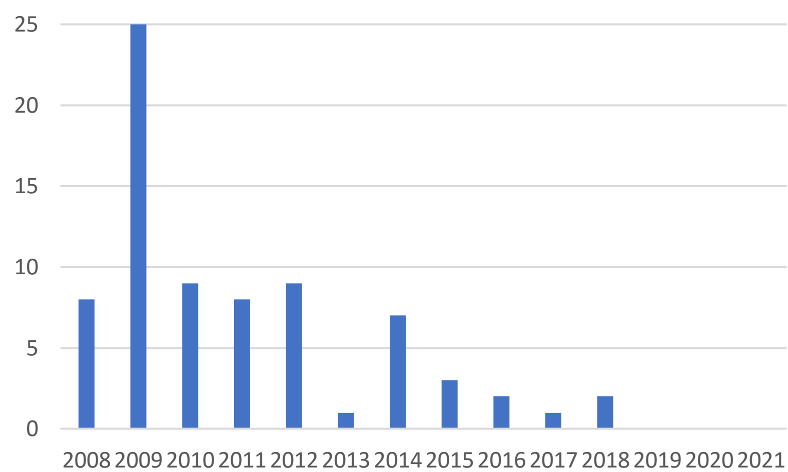
Figure 7.: BUX anomaly frequency chart at -0.018 threshold.



Source: own compilation based on MNB

To test whether the method provides a reasonable crisis forecasting alternative, the above-described procedure was conducted for the Ukrainian stock market index (UX) at -0.038 threshold Figure 8, the British Financial Times Stock Exchange (FTSE) at -0.051 threshold Figure 9 and the Russian stock market index (RTSI) at -0.048 Figure 10.

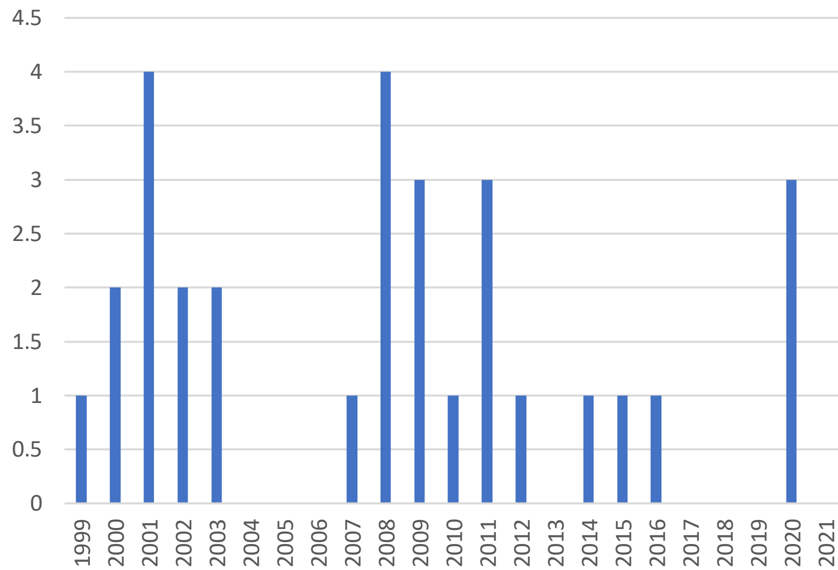
Figure 8.: UX stock market index at -0.038 threshold



Source: own compilation based on NBU

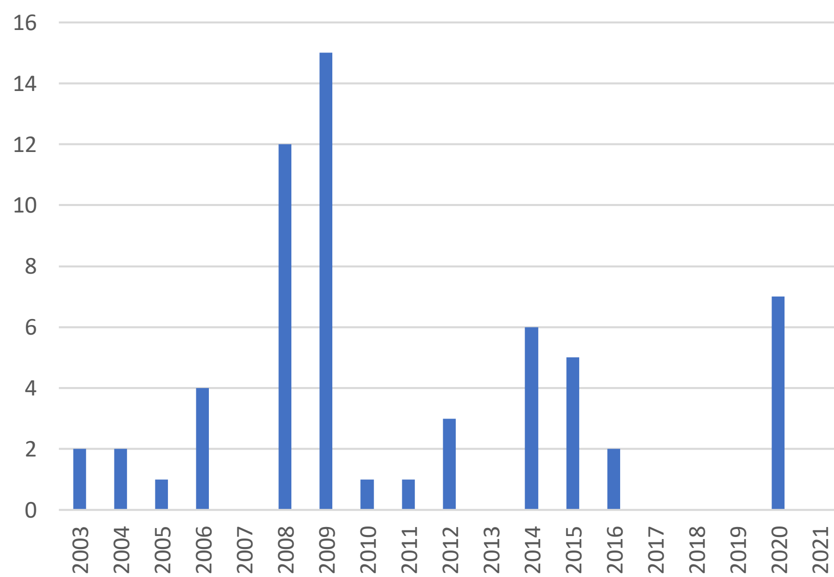
There are a number of reasons why the UX index histogram follows the specific shape depicted in Figure 8. Firstly, it is the factor of availability of data. The UX index has been introduced in early 2008, and therefore, captures a significantly narrower timeframe than other domestic indexes. Secondly, due to the radically different unstable political and economic frameworks, Ukraine suffered from more crises than any other country in our research.

Figure 9.: FTSE stock market index at -0.051 threshold.



Source: own compilation based on Yahoo Finance

Figure 10.: RTSI Russian stock market index at -0.048 threshold



Source: own compilation based on Moscow Stock Exchange

The conducted study on the effectiveness of the method in three EMs and one developed economy showed, that frequency analysis of negative overreactions in the stock market generates important information about the future state of the economic system and has predictive properties for the emergence of crisis phenomena in the economy. In contrast to traditional macroeconomic indicators - GDP, inflation index, etc., among stock market indicators, anomalous price fluctuations have a high predictive capacity and predictive properties. Their application was tested both to identify the economic crisis in Hungary, Ukraine, Russia and Great Britain in 2007-2009, as a consequence of the global financial crisis and the crisis in the economy of Ukraine and Russia in 2013-2015 and the mild recession in Hungary in 2011-2013. Based on the frequency analysis of the BUX index from 2001 to 2021, it has been determined that a rapid increase in the frequency of anomalous negative fluctuations can provide warning signals for forthcoming economic crises. The spike in the frequency of observed fluctuations in 2006 can relate to the increase of macroeconomic imbalances leading up to the 2008 currency, banking, and economic crises. While no anomalies below the -0.28 threshold were recorded in 2019 (in the year leading up to the 2020 COVID crisis), it can be argued that the spike of anomalous fluctuations in 2018 could have been a warning for rising macroeconomic imbalances. As for the RTSI spikes in price fluctuations in 2006 and 2012 could be interpreted as warning signals for the growth of economic imbalances and the subsequent 2008-2009 and 2014-2015 crises. On the basis of the frequency analysis of the UX index between 2008-2021, as a consequence of a smaller data sample, the relevant conclusions can’t be made. The increase of negative price fluctuations in 2010-2012 leading up to the 2013-2015 crises, however, proved to be correct in crisis forecasting. However, alike the BUX, a spike can be seen in 2018, that, again, can be interpreted as a warning for rising economic imbalances. The same case can’t be made for the FTSE index. No significant spikes were identified, which were leading up to the 2008, 2012 and 2020 recessions. A, yet, dubious conclusion can be therefore made, that the anomalous stock price fluctuation indicator works best for EMs, however this claim must be tested in further research papers.

3. Conclusion

The hypothesis tested in this section was whether the increase in the frequency of abnormal negative fluctuation in stock indexes indicates a forthcoming crisis or its intensification. While it has been determined that the phases of the crisis development correlate with the dynamics of changes in the frequency of anomalous negative fluctuations, the results of the yearly fluctuation distribution graphs of the analysed stock market indexes do not provide sufficient evidence to make them reliable crisis forecasting alternatives for developed economies such as Great Britain. The case whether the method provides a viable forecasting alternative for EMs must be tested on a larger data sample to provide statistically meaningful evidence. Furthermore, contrary to the opinion of contemporary literature on the method, it is believed that there is reasonable evidence to suggest that the time intervals for the final anomaly frequency tables should be in quarters, preferably in months, rather than years. The research on the methodology since its introduction in 2012 is still being developed.

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Appendix

Program Listing

```
pip install statsmodels


from statsmodels.tsa.stattools import grangercausalityt
ests
from statsmodels.tsa.stattools import adfuller
from scipy import stats
from statsmodels.tsa.api import VAR
from statsmodels.tools.eval_measures import rmse, aic
from statsmodels.tsa.stattools import kpss
import pickle
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

GDP = []
BUX = []

BUX = BUX[1:]
GDP = GDP[:-1]

ts_df = pd.DataFrame(columns=['GDP', 'BUX'], data=zip(G
DP, BUX))

ts_df

#for BUX instead of GDP plug in 'BUX'
result = adfuller(GDP)
print('ADF Statistics: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical values:')
for key, value in result[4].items():
    print('\t%s: %.3f' % (key, value))

#KPSS test
def kpss_test(GDP):
    print("Results of KPSS Test:")
    kpsstest = kpss(GDP, regression="ct", nlags="auto")
    kpss_output = pd.Series(
        kpsstest[0:3], index=["Test Statistic", "p-valu
e", "Lags Used"]
    )
    for key, value in kpsstest[3].items():
```

```
        kpss_output["Critical Value (%s)" % key] = valu
e
    print(kpss_output)
kpss_test(GDP)

print(f'Result: The series is {"not " if value < 0.05 e
lse ""}stationary')
kpss_test(BUX)

#Granger Causality test
gc_res = grangercausalitytests(ts_df, 3)
```